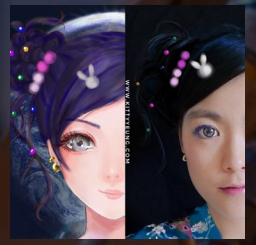
Introduction to Quantum Computing



Kitty Yeung, Ph.D. in Applied Physics

Creative Technologist + Sr. PM
Microsoft

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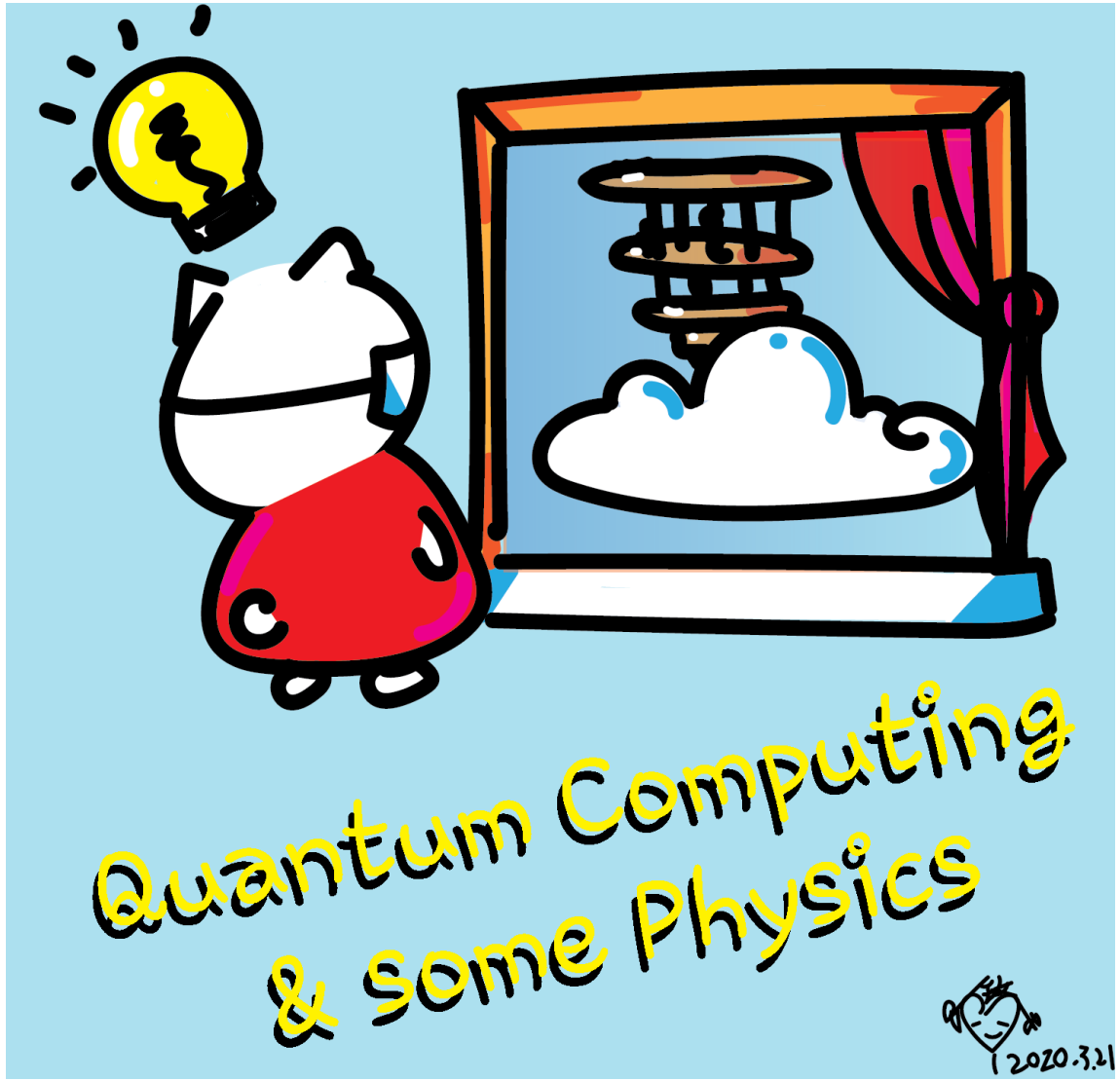
April 5, 2020
Hackaday, session 2

Class structure

- Comics on Hackaday – Introduction to Quantum Computing every Wed & Sun
- 30 mins every Sun, one concept (theory, hardware, programming), Q&A
- Contribute to Q# documentation http://docs.microsoft.com/quantum
- Coding through Quantum Katas https://github.com/Microsoft/QuantumKatas/
- Discuss in Hackaday project comments throughout the week
- Take notes



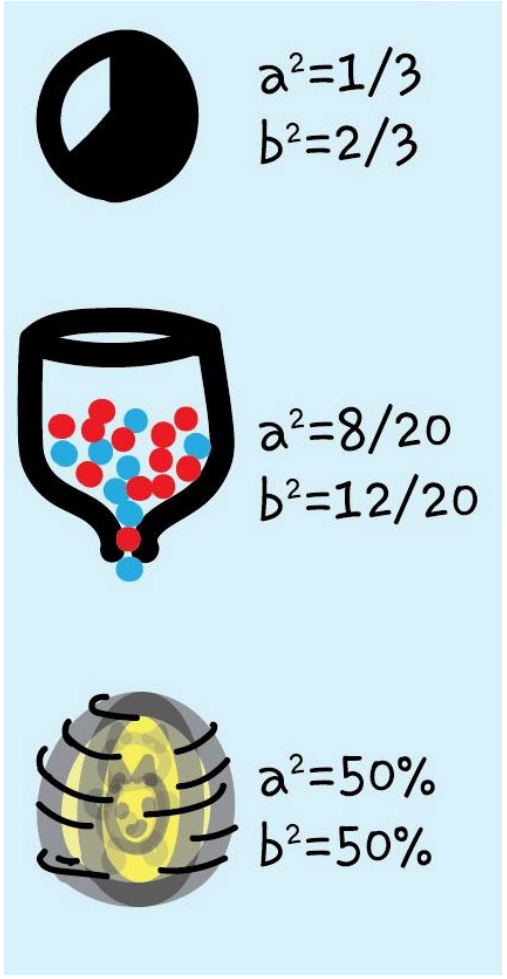
Reinforcement learning for natural intelligence

- Interactive class, feel free to ask questions
- Anything confusing? I'll try to explain a different way

Qubits & Superposition

$$|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix} = a|0\rangle + b|1\rangle$$

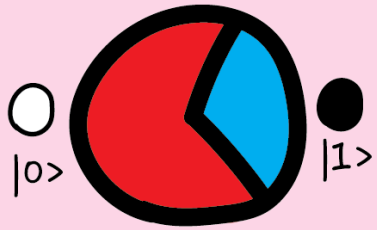
$$|a|^2 + |b|^2 = 1$$



2020.3.28.

A qubit system is all the possible configurations in superposition.

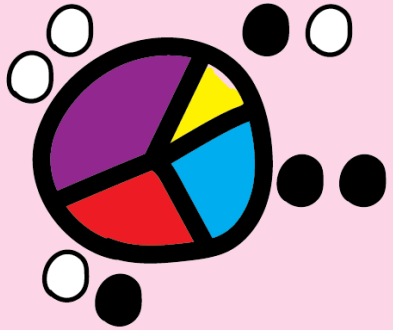
PIE CHART DENOTING PROBABILITY OF EACH CONFIGURATION



ONE QUBIT, TWO CONFIGURATIONS:

$a|0\rangle+b|1\rangle$

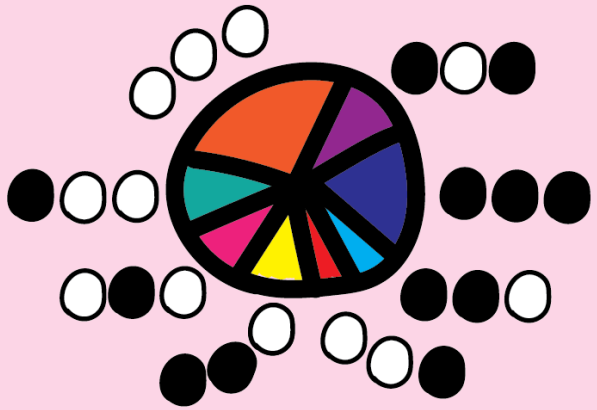
$a^2+b^2=1$ (total probability adds up to 1)



TWO QUBITS, FOUR CONFIGURATIONS:

$a|00\rangle+b|01\rangle+c|10\rangle+d|11\rangle$

$a^2+b^2+c^2+d^2=1$



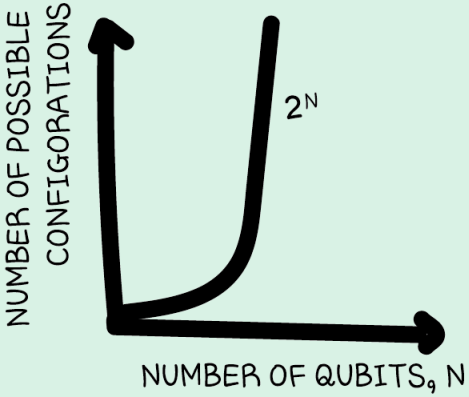
THREE QUBITS, EIGHT CONFIGURATIONS:

$a|000\rangle+b|001\rangle+c|010\rangle+d|100\rangle+e|110\rangle+f|101\rangle+g|011\rangle+h|111\rangle$

$a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2=1$

...

N qubits will have 2^N possible configurations in superposition!



Not only does the number of possible configurations grow exponentially with the number of qubits as 2^N, the number of possible combinations of amplitudes is infinite, as long as their squares – the probabilities – add up to 1.

THIS SYMBOL MEANS SUMMING ALL N TERMS FROM 1

$$|\psi\rangle = \sum_{i=1}^{N} c_i |\psi_i\rangle$$

AN N-QUBIT STATE

EACH POSSIBLE CONFIGURATION

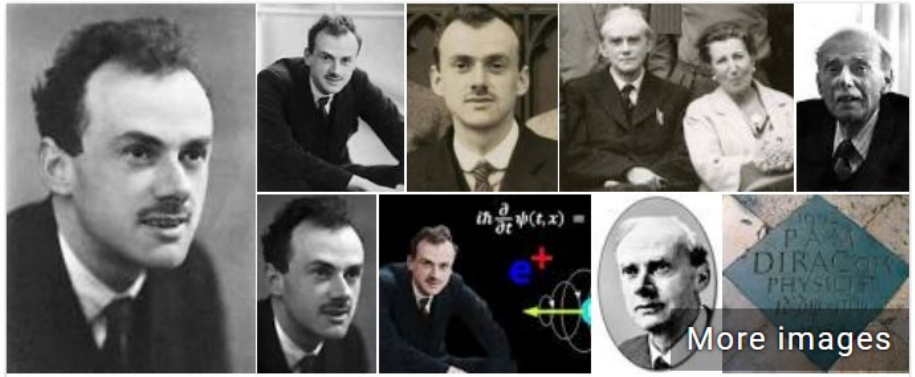
NATURE DOES PLAY DICE!!!

The amplitude ci = a_i b_i c_i d ...n
can be positive numbers 1_i $1/2_i$ $1/3_i$ 1/4 ...n
or negative numbers -1_i $-1/2_i$ $-1/3_i$ −1/4 ...n
(these are real numbers)
or imaginary numbers (+/−) i_i $1/2i_i$ $1/3i_i$ $1/4i$...ni
or 0.
In general they can be complex numbers
(with real and imaginary parts with
positive or negative signs)!

What's the consequence?

a|000>+b|001>+c|010>+d|100>+e|110>+f|101>+g|011>+h|111>

Dirac notation and wavefunction



Paul Dirac

Physicist

Paul Adrien Maurice Dirac OM FRS was an English theoretical physicist who is regarded as one of the most significant physicists of the 20th century. Dirac made fundamental contributions to the early development of both quantum mechanics and quantum electrodynamics. Wikipedia

Born: August 8, 1902, Bristol, United Kingdom

Died: October 20, 1984, Tallahassee, FL

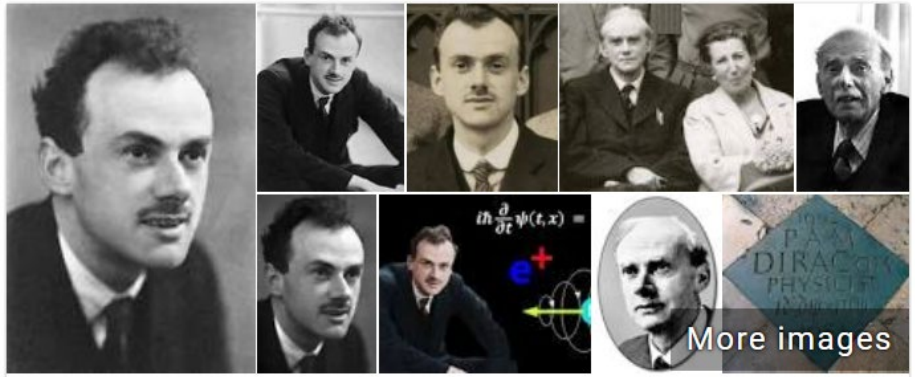
Field: Theoretical physics

Spouse: Margit Wigner (m. 1937–1984)

Schrödinger equation has the form of a wave equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\boldsymbol{r},t)+V(\boldsymbol{r},t)\Psi(\boldsymbol{r},t)=i\hbar\frac{\partial\Psi(\boldsymbol{r},t)}{\partial t}$$

Dirac notation and wavefunction



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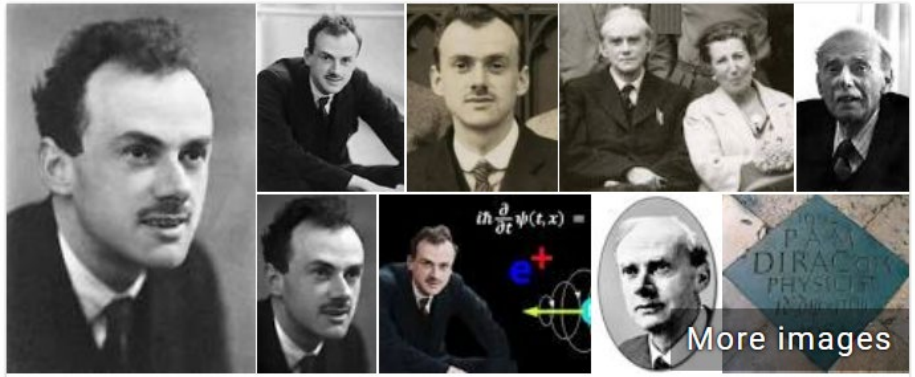
Therefore the solution is a linear combination Of all the possible wavefunctions

$$\psi(x)=\sum_i c_i\phi_i(x)$$

Dirac notation and wavefunction



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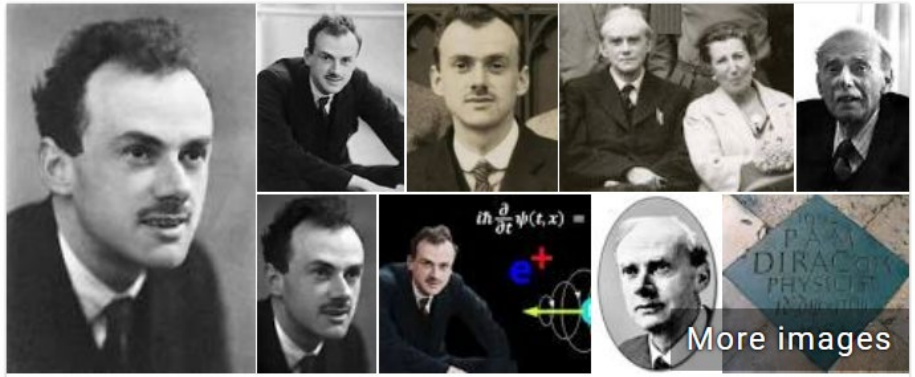
$$\psi(x)=\sum_i c_i\phi_i(x)$$

$$\int_{-\infty}^{+\infty}\phi_j{}^*(x)\ \psi(x)dx=\sum_i c_i\int_{-\infty}^{+\infty}\phi_j(x)^*\ \phi_i(x)dx=c_j\ .$$

In Dirac notation, $|\psi\rangle=\sum_i c_i|\phi_i\rangle$, where $c_j=\langle\phi_j|\psi\rangle$.

Dirac notation and wavefunction



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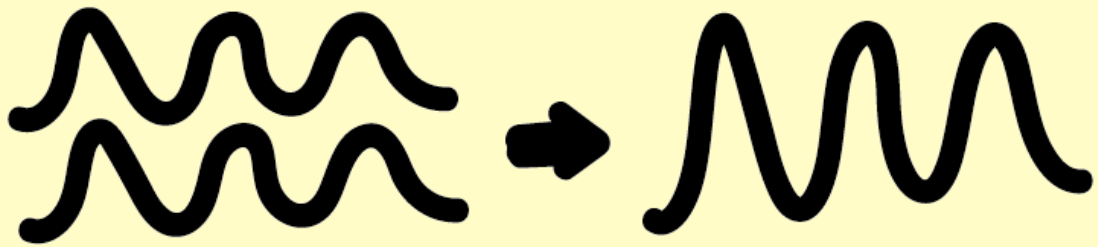
$|\Psi\rangle$ denotes "the state with wavefunction" $\Psi(\boldsymbol{r},t)$

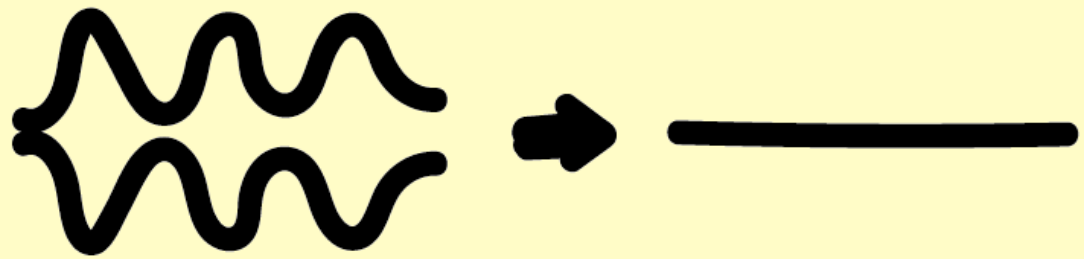
$$\Psi^*(\boldsymbol{r},t)=\langle\Psi|$$

$$\int_{-\infty}^{+\infty}\phi^*(x)\,\psi(x)\,dx\equiv\langle\phi|\psi\rangle$$

Our daily experience of amplitudes (like those of water waves, light waves, sound waves, etc.) has told us:



AMPLITUDES CAN ADD UP =
CONSTRUCTIVE INTERFERECE

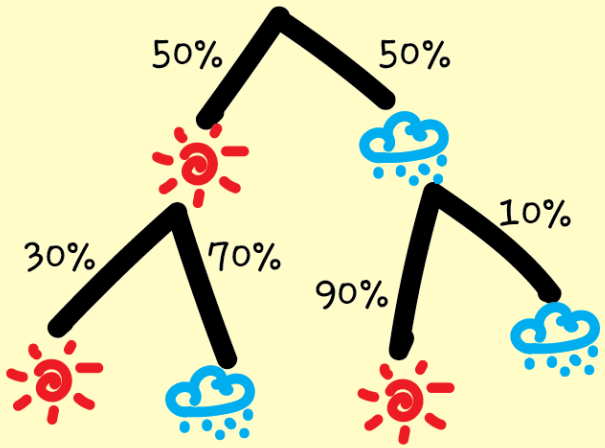


AMPLITUDES CAN CANCEL OUT =
DESTRUCTIVE INTERFERENCE

How likely will it be sunny the day after tomorrow?

CLASSICAL

USES PROBABILITY DIRECTLY

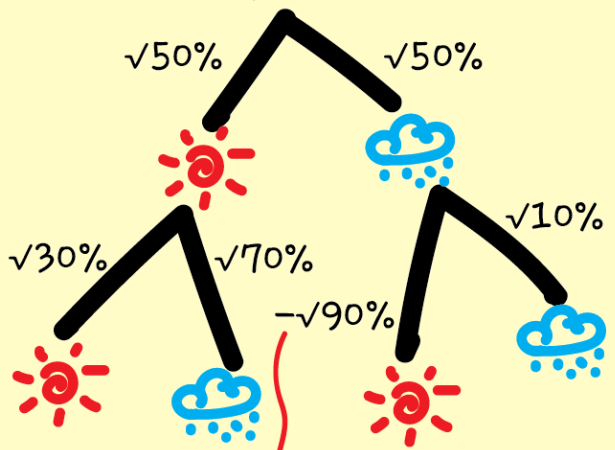


50%*30%+50%*90%=60%

Having more paths in classical case always leads to more likelihood.

QUANTUM

USES AMPLITUDE, AND CAN BE NEGATIVE

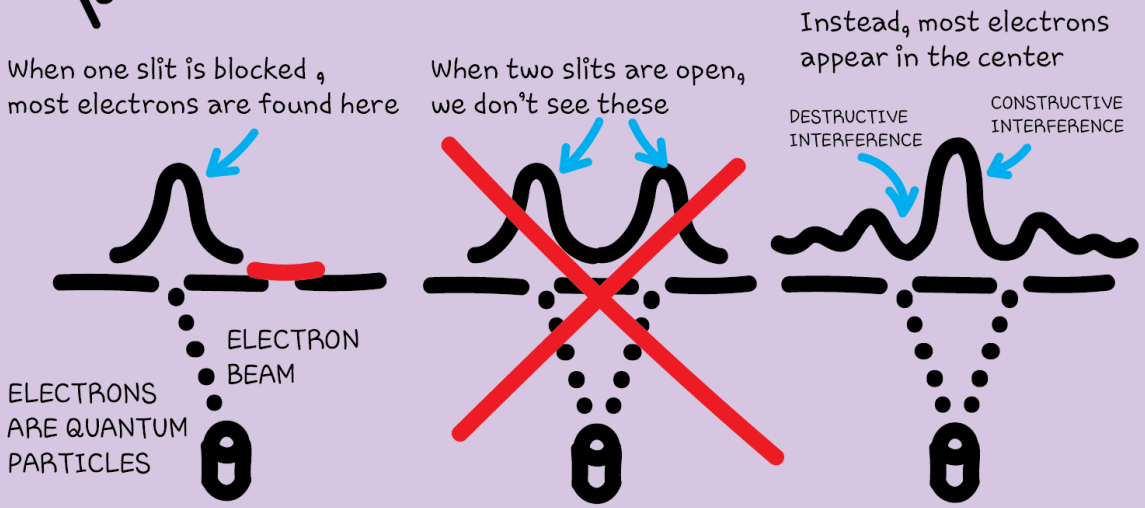


$|(\sqrt{50\%}*\sqrt{30\%}-\sqrt{50\%}*\sqrt{90\%})|^2=8\%$

But in quantum case, the 2nd path of having a sunny day destructively interferes with the 1st one, making it less likely.

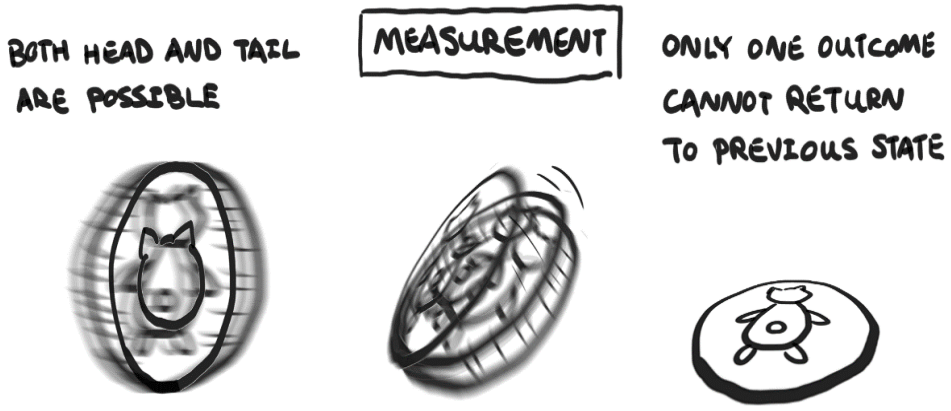
So, the things we observe (measure) are the results of interference. Possible results from constructive interference are more likely to be measured. The other possibilities cancel each other out through destructive interference.

The famous double-slit experiment is a direct manifestation of quantum interference.



Interference is one of the "strange" behaviours of quantum systems enabled by superposition. What else?

Measurement



Not reversible

$$|\psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

$$P = |c_{00}|^2 + |c_{01}|^2$$ If first qubit is 0

$$|\psi'\rangle = \frac{c_{00}|00\rangle + c_{01}|01\rangle}{\sqrt{P}}$$ After measurement

Generalized probability theory

$$\sum_i p_i = 1$$

1-norm
Classical

$$\sum_i |a_i|^2 = 1$$

2-norm
Quantum mechanical

Amplitude can be positive, negative or complex

Scott Aaronson

American computer scientist

Scott Joel Aaronson is an American theoretical computer scientist and David J. Bruton Jr. Centennial Professor of Computer Science at the University of Texas at Austin. His primary areas of research are quantum computing and computational complexity theory. Wikipedia

Born: May 21, 1981 (age 37 years), Philadelphia, PA

Nationality: American

Spouse: Dana Moshkovitz

Books: Quantum Computing Since Democritus

Known for: PostBQP, P versus NP problem, Boson sampling

Education: Cornell University, University of California, Berkeley

2-norm Vs 1-norm
https://www.scottaaronson.com/democritus/lec9.html

To read more rigorous mathematical derivations of the axioms in modern quantum theory:

- https://arxiv.org/abs/quant-ph/0101012
- https://arxiv.org/abs/1011.6451
- https://arxiv.org/abs/quant-ph/0104088

Future topics